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Analysis of recent developments in linear time base generators

Stout, Fred Elmer, Jr.

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ANALYSIS OF RECENT DEVELOPMENTS IN LINEAR TIME BASE GENERATORS

BY

F. E. STOUT, JR.

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Fred Elmer Stout, Jr.
Lieutenant, United States Navy

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PREFACE

This thesis is based on library research and practical knowledge acquired in the laboratory of the Electronics Division of the Westinghouse Corporation, Baltimore, Maryland, where from 11 January to 6 April, 1951, the author worked with circuits of the nature discussed, in connection with automatic tracking. It is written to provide an integrated, detailed treatment of recent developments in linear time base generators.

F. E. STOUT, JR.

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TABLE OF SYMBOLS AND ABBREVIATIONS

E_{bb} - Plate supply voltage.

E_{cc} - Grid supply voltage.

E_{sc} - Screen supply voltage.

E_{SUP} - Suppressor biasing voltage.

e_b - Instantaneous plate voltage.

e_g - Instantaneous grid voltage.

v_{sc} - Instantaneous screen voltage.

v_{SUP} - Instantaneous suppressor voltage.

v_K - Instantaneous cathode voltage.

e_{in} - Instantaneous input voltage to amplifier.

R_L - Plate load resistor.

R - Resistor from grid to E_{cc}.

R_K - Cathode resistor.

C - Grid-plate coupling condenser.

C_s - Distributed capacity from plate to ground.

i_b - Plate current.

i_c - Current through C.

i_L - Current through R_L.

i_s - Current supplied by C_s.

I_L - Current through plate load inductance.

e_{fb} - Voltage fed back from plate to grid.

A - Absolute value of amplifier gain.

G - Absolute value of amplifier gain with feedback.

Z - Output impedance at plate of amplifier without feedback.

V_s - Control voltage.

TABLE OF SYMBOLS AND ABBREVIATIONS
(continued)

$\mathcal{L}[\quad]$ - LaPlace transform of $[\quad]$.

$\mathcal{L}^{-1}[\quad]$ - Inverse LaPlace transform of $[\quad]$.

CHAPTER I

INTRODUCTION

An important problem in modern electronic circuits is the generation of a linear time base. The solution to this problem depends upon the performance demanded of the generator. Most applications are continuously requiring better linearity, faster switching and recovering action, less cost and more stability.

Until recently, linear time base generators depended mainly upon the multivibrator. While this circuit had inherent disadvantages for certain applications, such as a high output impedance, it satisfied the demands associated with early radar development. The literature shows that a circuit based on multivibrator action can deliver an output discontinuity whose position in time within several per cent is directly proportional to an input voltage. A rigorous analysis of multivibrator action is available which can be used to improve the performance of multivibrator circuits(12). From a practical viewpoint, however, the multivibrator is critically dependent upon certain important factors. The rigorous analysis mentioned above presupposes absolute identity of the tubes. In practise, this specification is never met and tube replacement can change the transfer characteristic of the circuit by 10 per cent. Another factor is the necessity for good voltage regulation. A variation in the line voltage of 10 per cent can alter the transfer characteristic by 2 per cent(3,4).

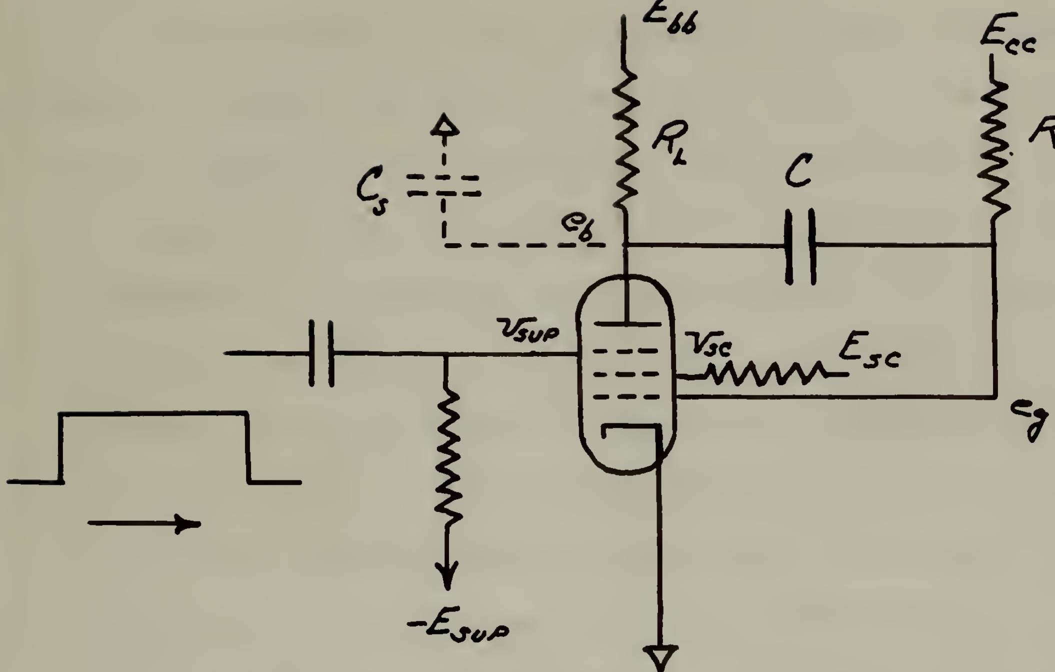
It became apparent during World War II that any attempt to adapt the multivibrator linear time base generator to the shrinking tolerances in the specifications was impractical. Accordingly, both England and the United States began searching for new solutions to the problem. As might be expected, each country investigated the use of feedback circuits for their inherent stability and low output impedance. Ultimately England developed a family of circuits based upon the Miller sweep generator. Simultaneously the United States developed a family of circuits based upon the bootstrap principle. In the chapters which follow these two types of circuits will be discussed and compared, with emphasis on the principles involved rather than the many specific solutions which have been developed.

CHAPTER II

THE MILLER SWEEP GENERATOR

The basic circuit of a Miller sweep generator is shown in Figure 1. Quiescently, the suppressor of the pentode is biased sufficently negative to cut off plate current. Because R is returned to the positive voltage E_{cc}, current is initally flowing from E_{cc} to ground through R and the grid-cathode pathway. Thus the grid bias is approximately zero and the screen is conducting heavily. The plate voltage is E_{bb} and the voltage across C must be E_{bb}.

A positive input gate voltage on the suppressor, large enough to permit plate current to flow, may be regarded as a switch which turns the pentode amplifier on and off. At the instant the amplifier is turned on, almost all of the plate current will be furnished from the charge on the distributed capacity, C_s, from plate to ground, and hence we may consider the voltage across C as remaining constant at this time. The first incremental drop in plate voltage associated with the initial increment of plate current will therefore be effectively coupled by C to the grid, dropping the grid below zero and opening the grid-cathode pathway. With a rapid succession of such increments, the grid quickly attains the negative bias necessary to satisfy the equilibrium conditions of the amplifier for the plate load resistor R_L. At this point the plate has dropped a few volts, almost abruptly; the grid has dropped below zero by the same amount; and the total space



BASIC MILLER SWEEP GENERATOR

FIGURE 1

current in the pentode has been reduced to a low value by the negative grid bias, with most of this small current flowing from the plate (through R_L), and the remainder flowing from the screen.

Subsequently, three actions take place concurrently to give a linear rundown at the plate. First, the grid immediately begins rising toward E_{cc}; next, each positive-going increment at the grid results in an amplified negative-going increment at the plate, which is coupled by C back to the grid; and simultaneously C is discharging. As a result of these actions, the grid tends to remain at a relatively constant negative voltage. The current through R and C is therefore relatively constant, and to the extent that this current is constant, the voltage across C is a negative-going sawtooth with respect to the grid voltage. Further, to the extent that the grid voltage is constant, the voltage from the plate of the amplifier to ground must also be a negative-going sawtooth.

If the duration of the positive gate on the suppressor is long enough, the plate voltage will eventually reach the knee of the e_b - i_b characteristics of the pentode, and will not be able to decrease further for the small positive-going increments in the grid voltage. At this instant the plate is said to "bottom", and immediately the grid, in the absence of any negative feedback through C, begins an unrestrained rise toward E_{cc}. The grid soon reaches zero, however, and is held at this potential by conduction through the grid-cathode pathway. The grid then continues to ride at zero and the plate at

the bottomed voltage as long as the amplifier remains turned on.

When the input gate on the suppressor ends and the amplifier is turned off, the plate voltage rises exponentially to E_{bb} with a time constant determined by R_L, C, and the distributed capacity in the plate circuit.

Should the switching gate terminate before the plate voltage reaches the knee of the e_b - i_b characteristics, the linear rundown of the plate will of course be arrested at this instant, and the plate and the grid will immediately begin rising to their quiescent voltages, with the rise of the plate voltage to E_{bb} again determined by the product of R_L, C, and the distributed capacity in the plate circuit. In this case the grid will jump to zero almost instantaneously.

Let us now examine the benefits that have evolved from the use of negative feedback in this circuit. Without feedback,

$$e_b = -A e_{in} \quad . \tag{1}$$

(When the amplifier is turned on, the input voltage to the grid is obviously a step voltage of magnitude E_{cc}.)

$$e_{fb} = \beta e_b = -\beta A e_{in} \quad , \tag{2}$$

where β is the feedback factor.

$$\therefore \; e_g = e_{in} - e_{fb} = e_{in}(1+\beta A) \quad . \tag{3}$$

For the gain with feedback,

$$G = \frac{e_b}{e_g} = \frac{A}{1+\beta A} \quad . \tag{4}$$

$$\frac{dG}{dA} = \frac{1}{(1+\beta A)^2} \quad . \tag{5}$$

$$\therefore \quad \frac{dG}{G} = \frac{1}{1+\beta A} \frac{dA}{A} \quad . \tag{6}$$

Thus, if A, the gain of the amplifier without feedback, were to change by some amount, the change in the gain of the amplifier with the negative feedback indicated would be only $\frac{1}{1+\beta A}$ as much. Hence, the Miller sweep generator is very stable, and the stability improves with increasing gain.

Another advantage resulting from the use of negative feedback is a large reduction in the output impedance at the plate of the amplifier. If this output impedance without feedback is Z, and if an external voltage source of zero internal impedance is adjusted for one volt and applied to the output terminals of the amplifier, $\frac{1}{Z}$ amps will be drawn from the source. With negative feedback in the circuit, however, a component of voltage $A\beta$ appears across the output terminals and draws $\frac{A\beta}{Z}$ amps from the source. The total current drawn from the one volt source will be $\frac{1}{Z} + \frac{A\beta}{Z}$ or $\frac{1+A\beta}{Z}$ amps. The output impedance with negative feedback is therefore $\frac{Z}{1+A\beta}$, which can be made very low with high gain.

Investigation of the input impedance at the grid of the amplifier is also interesting. If e_g rises by an increment

Δe_g, the plate voltage falls by an increment $A\,\Delta e_g$, and the current passing through C will be $\frac{\Delta e_g + A\,\Delta e_g}{Z_c}$ or $\frac{\Delta e_g}{Z_c}(1+A)$. The effective input impedance at the grid is therefore $\frac{Z_c}{1+A}$. In other words, the input capacitance is $C(1+A)$. This is really a statement of the principle from which the Miller sweep generator gets its name. The Miller Effect equation states that for an amplifier,

$$C_{in} = C_{gk} + C_{gp}(1 + A\cos\theta) , \qquad (7)$$

where θ is the phase angle of the load. In this case C_{gp} is large in comparison with C_{gk}, because we have deliberately added a capacitance between the grid and the plate. For a resistive load, $\cos\theta$ is unity, and hence $C_{in} = C(1+A)$, which is the result achieved above.

When the generator is turned on, then, the grid commences to rise toward E_{cc} with a time constant of approximately ARC seconds. If the gain of the amplifier is high, the change in grid voltage during the rundown at the plate is very small. If the gain were infinite, the grid voltage would of course remain constant.

We might expect from the foregoing that the plate voltage would approach $-A\,E_{cc}$ with the same time constant. A more detailed analysis of the circuit brings out this fact. Using LaPlace Transform notation,

$$e_b(s) = e_g(s) - \frac{1}{Cs} I_R(s) . \qquad (8)$$

If the gain of the amplifier is assumed constant,

$$e_g(s) = -\frac{e_b(s)}{A} \quad . \tag{9}$$

Then

$$I_R(s) = \frac{E_{cc}(s) - e_g(s)}{R}$$

$$= \frac{E_{cc}(s) + \frac{e_b(s)}{A}}{R} \quad . \tag{10}$$

$$\therefore\ e_b(s) = -\frac{e_b(s)}{A} - \frac{1}{Cs}\frac{E_{cc}(s)}{R} - \frac{1}{Cs}\frac{e_b(s)}{AR} \quad . \tag{11}$$

From this expression we get

$$e_b(s) = \frac{-A E_{cc}(s)}{RC(A+1)} \quad \frac{1}{\left[s + \frac{1}{RC(A+1)}\right]} \quad , \tag{12}$$

whence,

$$\mathcal{L}e^{-1}\left[e_b(s)\right] = e_b(t) = -A E_{cc}\left[1 - e^{\frac{-t}{RC(A+1)}}\right] . \tag{13}$$

Converted to series form,

$$e_b = \frac{-A E_{cc}}{(A+1)RC}\left[t - \frac{t^2}{2(A+1)RC} + \text{-----}\right] . \tag{14}$$

The departure from linearity of the plate voltage is represented by all terms of the series after the first term. The largest of these subsequent terms is the second term. With a high gain amplifier, the product $(A+1)RC$ can be made very large, so that

$$e_b \doteq \frac{-AE_{cc}}{(A+1)RC}\, t \quad . \tag{15}$$

Hence, e_b approaches $-A\,E_{cc}$ with the time constant $(A+1)RC$, and, with small error, varies linearly with time.

Thus far, however, we have not considered the current through R_L or the effect of the pentode characteristics. For small variations in the grid voltage of a pentode(1),

$$i_b = g_m \left(e_g + \frac{v_{sc}}{\mu_{sc}} + \frac{e_b}{\mu} \right) . \tag{16}$$

In this case,

$$e_g = E_{bb} - R_L i_L - v_c \quad . \tag{17}$$

$$\therefore \; i_b = g_m \left[\left(E_{bb} - R_L i_L - v_c \right) + \frac{v_{sc}}{\mu_{sc}} + \frac{E_{bb} - R_L i_L}{\mu} \right] . \tag{18}$$

Now $i_b = i_L + i_s + i_c$, where i_s is the current supplied by the distributed capacitance.

Solving for i_L ,

$$i_L = \frac{g_m\left(E_{bb} - v_c + \frac{v_{sc}}{\mu_{sc}} + \frac{E_{bb}}{\mu}\right) - i_c - i_s}{1 + g_m R_L + \frac{g_m R_L}{\mu}} \; . \tag{19}$$

Since the linear rundown of the plate voltage occurs above the knee of the pentode $e_b - i_b$ characteristics, g_m, μ, v_{sc}, and μ_{sc} remain relatively constant. We have shown $i_c = i_R = \frac{E_{cc} - e_g}{R}$ also to be relatively constant during the plate rundown, so that $\frac{dv_c}{dt} = \frac{i_c}{C} = \frac{E_{cc} - e_g}{RC}$. i_s is negligible compared with i_c, and E_{bb} is a constant.

$$\therefore \frac{d\,i_L}{dt} = \frac{-g_m\left(\frac{E_{cc} - e_g}{RC}\right)}{1 + g_m R_L + \frac{g_m R_L}{\mu}} \; , \tag{20}$$

and $$\frac{de_b}{dt} = R_L \frac{di_L}{dt} = \frac{-g_m R_L}{1 + g_m R_L\left(1 + \frac{1}{\mu}\right)} \left(\frac{E_{cc} - e_g}{RC}\right) . \tag{21}$$

Further, if $\mu \gg 1$ and $g_m R_L \gg 1$,

$$\frac{de_b}{dt} = -\left(\frac{E_{cc} - e_g}{RC}\right) = K \; , \tag{22}$$

and $e_b = Kt + B$, the equation of a straight line.

It is apparent from the foregoing that the Miller sweep generator has several features important in a linear time base generator. It is a very stable circuit, and at a low output

impedance it generates a highly linear waveform.

CHAPTER III

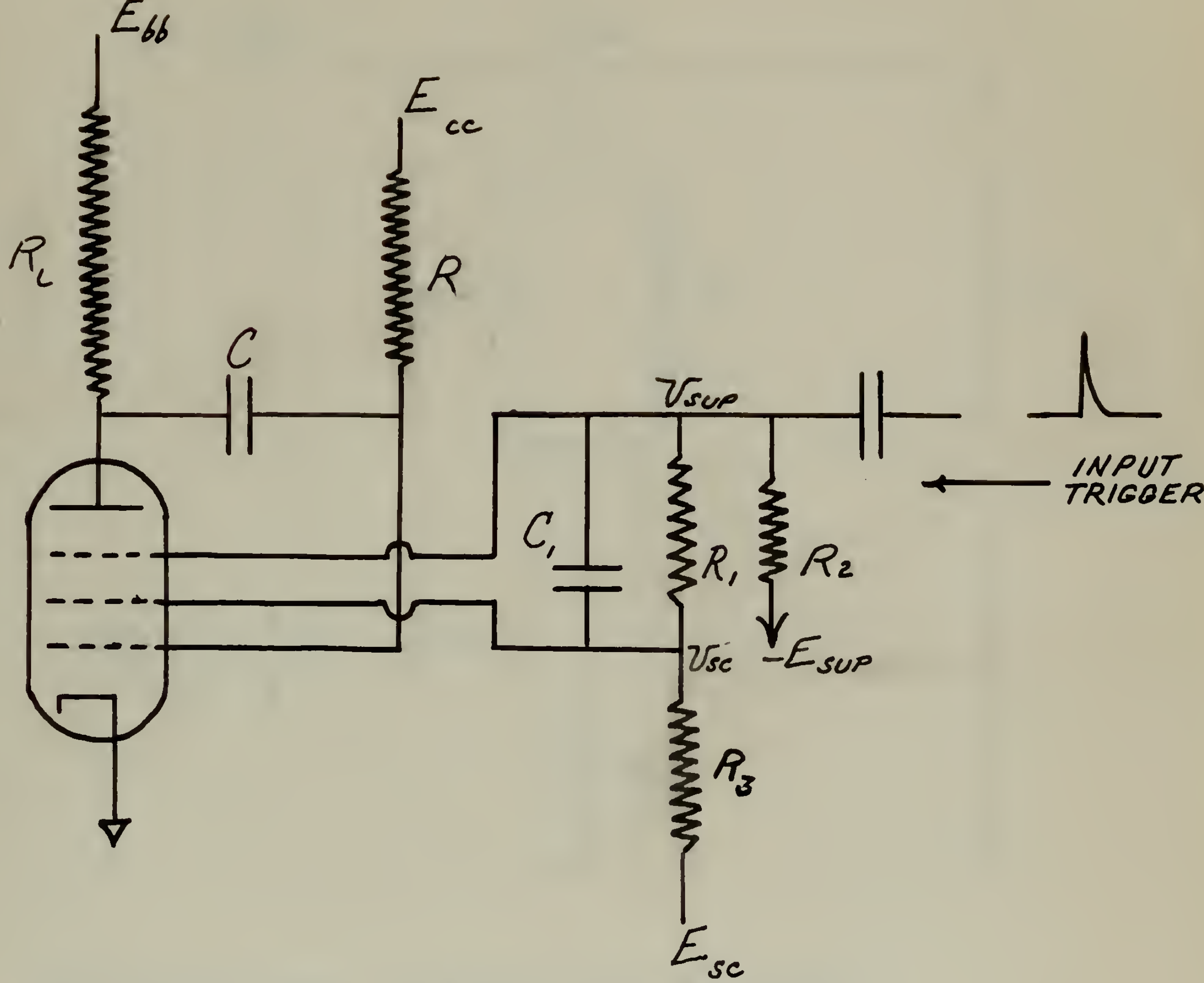
CIRCUITS BASED ON THE MILLER SWEEP GENERATOR

When the need for better linear time base generators arose during World War II, England began developing a family of circuits based upon the Miller sweep generator. These circuits are now known as the phantastron, sanatron, and sanaphant, and differ principally in the manner in which the amplifier is turned on and off. The input voltage to this type of circuit is in all cases a trigger voltage, instead of a gate voltage. The gating is accomplished within the generator.

There are two versions of the phantastron: the screen-coupled phantastron (which the British choose to call the Miller transitron), and the cathode-coupled phantastron.

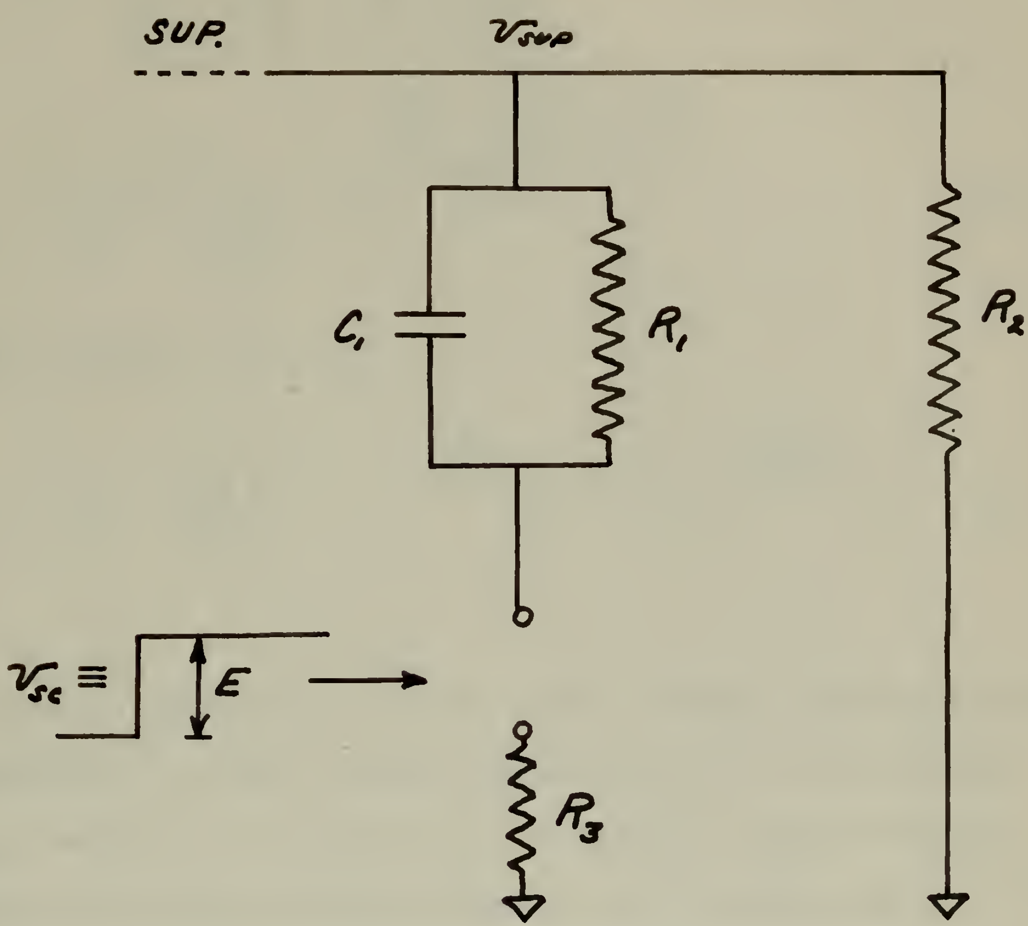
The screen-coupled phantastron is represented in Figure 2. Before the input trigger is applied, the grid is at zero, the screen is conducting heavily, and plate current is cut off by the negative bias, $-E_{SUP}$, on the suppressor. Application of the input trigger to the suppressor causes plate current to flow. The resultant plate voltage drop is coupled by C to the grid, reducing the space current drastically. The screen therefore jumps almost to E_{sc}. Since the jump in screen voltage is almost instantaneous, we may regard the jump as a step voltage of amplitude E, applied to the RC network of Figure 3.

Analysis of this circuit indicates that the resulting



SCREEN-COUPLED PHANTASTRON

FIGURE 2



SWITCHING CIRCUIT OF THE SCREEN-COUPLED PHANTASTRON

FIGURE 3

suppressor voltage is of the form

$$v_{SUP} = \frac{\mathcal{E}}{\left(1 + \frac{R_3}{R_2}\right)} \frac{R_2}{\left(\frac{R_2}{R_1} + \frac{R_3}{R_1} + 1\right)R_1} \left(1 - e^{\frac{-\left(\frac{R_2}{R_1} + \frac{R_3}{R_1} + 1\right)t}{R_2 C_1}}\right)$$

$$+ \frac{\mathcal{E}}{\left(1 + \frac{R_3}{R_2}\right)} e^{\frac{-\left(\frac{R_2}{R_1} + \frac{R_3}{R_1} + 1\right)t}{R_2 C_1}} \quad . \qquad (23)$$

If $R_1 >> R_2 >> R_3$,

$$v_{SUP} \doteq \mathcal{E} \frac{R_2}{R_1}\left(1 - e^{-\frac{t}{R_2 C_1}}\right) + \mathcal{E} e^{-\frac{t}{R_2 C_1}} \quad . \qquad (24)$$

The rising characteristic of the first voltage component tends to compensate for the falling characteristic of the second voltage component, so that v_{SUP} is closely a step voltage.

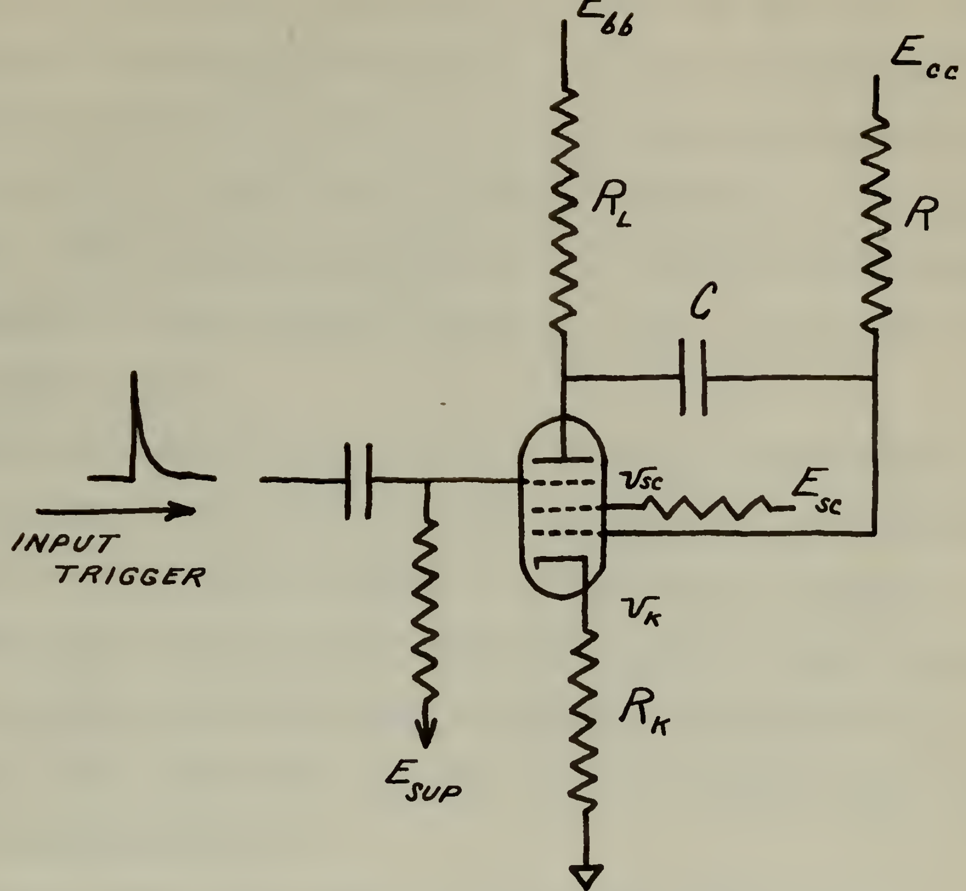
When the plate rundown bottoms and the grid commences to rise, space current increases and the screen voltage begins to fall. Almost instantaneously the screen voltage reaches its quiescent value before the trigger was applied. This action is equivalent to applying a negative step voltage to the circuit of Figure 3, and hence, by the same analysis as before, a negative approximate step voltage is applied to the suppressor, and the amplifier is abruptly turned off, and held in this condition by $-\mathcal{E}_{SUP}$. Internal gating of the basic Miller sweep generator is accomplished in this manner.

In the cathode-coupled phantastron, gating is effected through the use of a cathode resistor, as shown in Figure 4. Quiescently, the grid is at the cathode voltage V_K . E_{SUP} is adjusted to a value which will bias the suppressor with respect to the cathode sufficiently to cut off plate current. The plate voltage is E_{bb}, and the screen is at a low voltage since almost all of the space current is going to the screen.

Upon application of the trigger input, plate current begins to flow and the plate voltage drops. The drop in plate voltage is coupled to the grid through C, reducing the total space current. The cathode voltage therefore drops and the suppressor is no longer biased with respect to the cathode sufficiently to keep the plate current cut off. Since this action is cumulative and almost instantaneous, a positive step voltage is in effect being applied to the suppressor. The switch-on action of the amplifier ends with the grid at such a negative bias with respect to the cathode that the space current is almost cut off. The screen has therefore almost instantaneously passed from a state of high conduction to a state of low conduction, and hence has quickly jumped practically to E_{SC}.

At the end of the switch-on action the linear rundown of the plate commences, for the amplifier has been turned on and quiescent conduction through the grid-cathode pathway has been interrupted.

After the plate voltage bottoms, the grid rises toward E_{cc} with a time constant of RC. The space current increases



CATHODE-COUPLED PHANTASTRON

FIGURE 4

sharply, with most of the current going to the screen. v_K increases to the point where the suppressor is so biased with respect to the cathode that plate current is cut off. This switch-off of the amplifier occurs rapidly, and subsequently the plate rises to E_{bb} with a time constant determined by the product of R_L, C, and the distributed capacity in the plate circuit. While rising toward E_{cc}, the grid of course begins to conduct at that potential where $e_g = v_K$, and hence levels off at this point.

The main requirement for both the screen-coupled and cathode-coupled phantastrons is a pentode with good suppressor control over the plate current. In place of a pentode a pentagrid tube may be used, with grids 2 and 4 replacing the pentode screen, and with grid 3 replacing the pentode suppressor. The requirement in this case is a sharp $e_{g_3} - i_b$ cut-off characteristic.

It has been shown that for the basic Miller sweep generator circuit,

$$e_b \doteq \frac{E_{cc}}{RC} t .$$

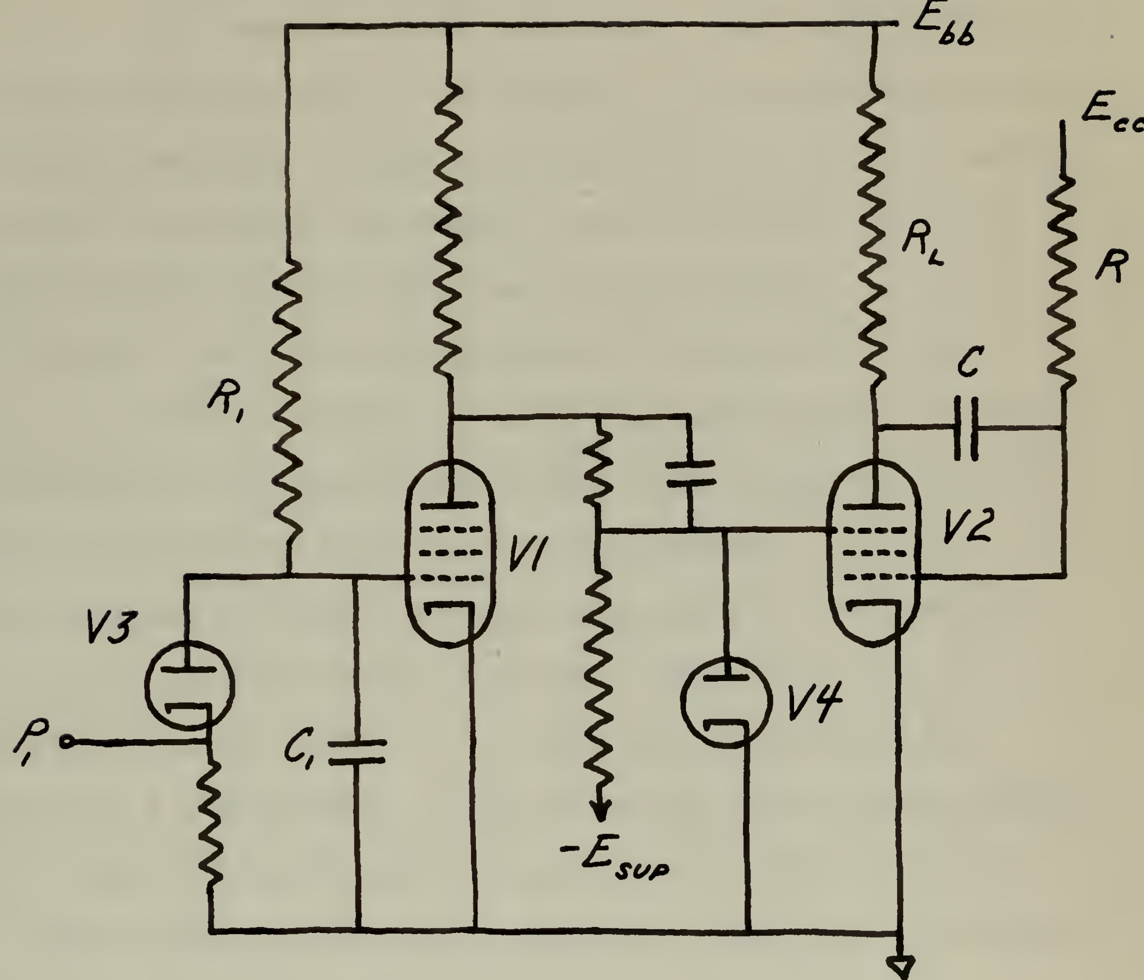
If E_{cc} is fixed in value, R and/or C must be made smaller for faster sweep speeds. As R is made smaller, however, the quiescent grid-cathode voltage becomes more positive, and more space current flows. It is therefore apparent that very fast sweep speeds must be limited by the amount of dissipation that the screen can tolerate. Since the tubes suitable for use in

screen-coupled and cathode-coupled phantastrons rarely have high screen dissipation ratings, the maximum sweep speeds that these phantastrons can produce are limited.

If very fast sweep speeds are required, the tube employed must be able to handle a high screen dissipation. But this type of tube rarely has good suppressor control over the plate current, and hence some means must be provided to gate the suppressor effectively. The sanatron and the sanaphant were designed to incorporate this requisite.

The basic sanatron circuit is represented in Figure 5. V1 is the gating switch and V2 is the Miller sweep generator. Quiescently, V1 is conducting heavily with its plate voltage low. The suppressor of V2 is held well below plate current cutoff by the voltage divider relationship indicated. The gating action begins when a negative input trigger applied at P1 cuts off V1. Almost instantaneously the plate of V1 rises to a high potential. This jump in plate voltage is applied to the suppressor of V2 through the voltage divider, which is so arranged that the suppressor would be driven positive if it were not for the diode V4. With the amplifier V2 thus turned on, the Miller sweep generator is activated.

Meanwhile, the grid of V1 commences to rise toward E_{bb} with time constant R_1C_1. When the grid cutoff voltage is reached on this upward swing, V1 conducts and its plate voltage drops abruptly. With this drop in plate voltage applied through the voltage divider to the suppressor of V2, the V2 amplifier is turned off and the sweep action at the



BASIC SANATRON CIRCUIT

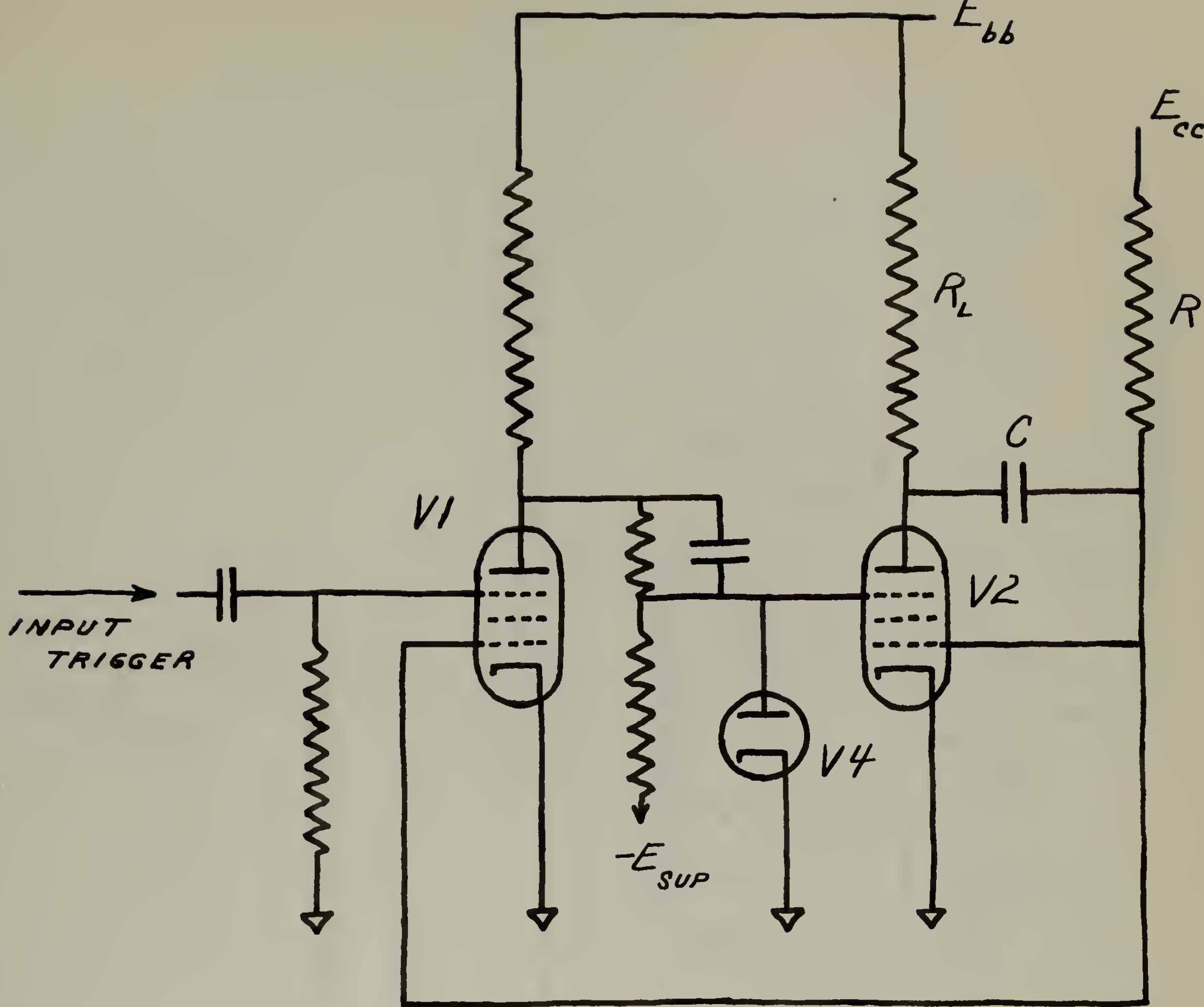
FIGURE 5

plate of V2 is terminated.

A common variation of the sanatron is contained in Figure 6. The operation of this circuit is identical to the operation of the sanatron just discussed. The only difference between the two circuits is in the use of the negative gate at the grid of the Miller sweep generator to maintain the gating pentode cut off during the sweep, and in the application of the input trigger to the suppressor of V1 instead of to the grid. In this case the gating action ends when the sweep action of the Miller generator naturally terminates, whereas in the previous case the gating action ends when the grid of V1 reaches the cutoff point of its own accord.

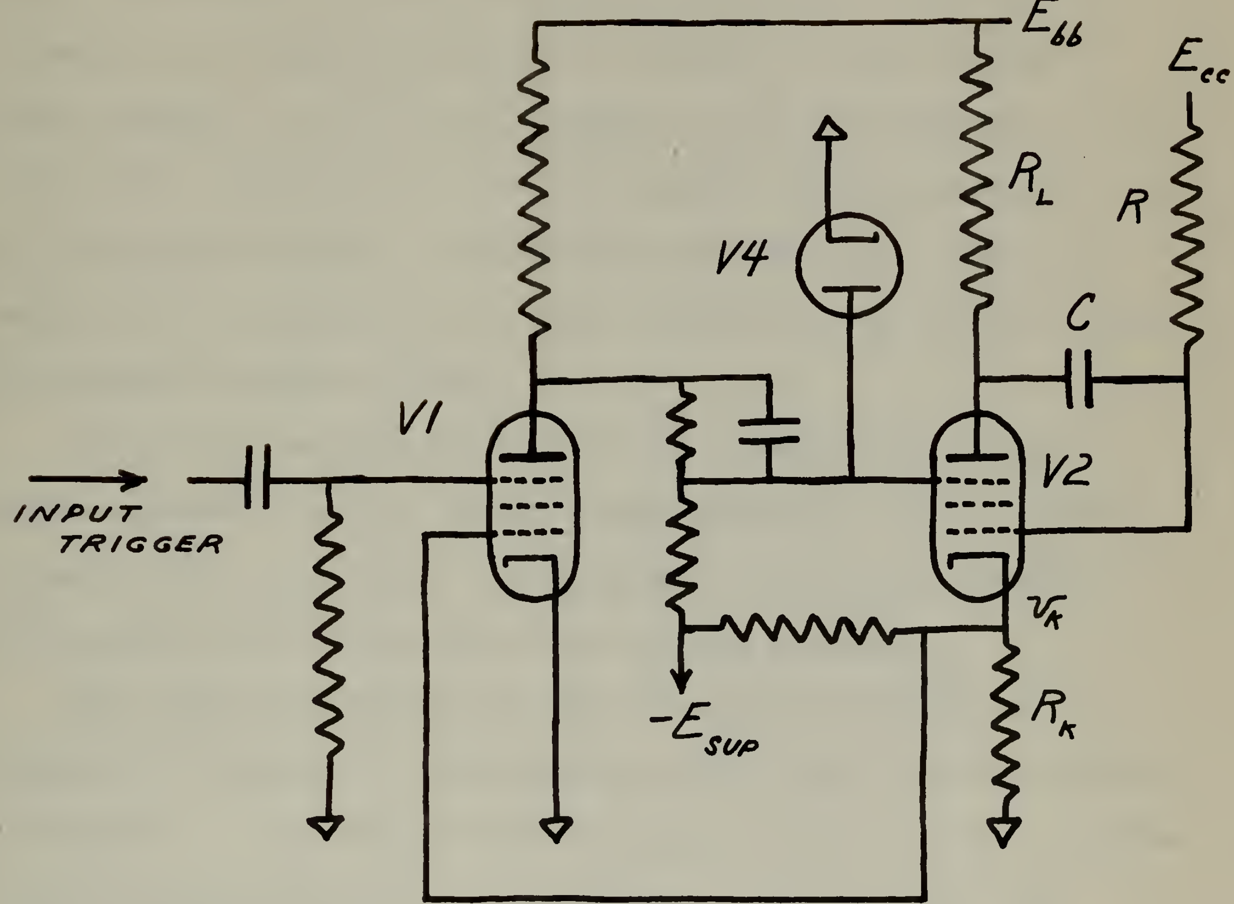
Another circuit which permits the use of a tube with a rugged screen grid and less suppressor control is the sanaphant, shown in Figure 7. As the name implies, the sanaphant is a combination of the sanatron and the phantastron, where the word "phantastron" is used in the British sense to mean a self-gating Miller sweep generator employing a cathode resistor.

Again, V1 is the gating pentode. In the quiescent state, V1 is conducting heavily with a low plate voltage which holds the suppressor of V2 below cutoff, and the cathode of V2 is riding at some potential close to ground. Application of a negative input trigger to the suppressor of V1 causes the plate of V1 and hence the suppressor of V2 to jump by an amount sufficient to turn on V2 as an amplifier. But the resultant drop in the grid voltage of V2 reduces the total



COMMON VARIATION OF THE BASIC SANATRON CIRCUIT

FIGURE 6



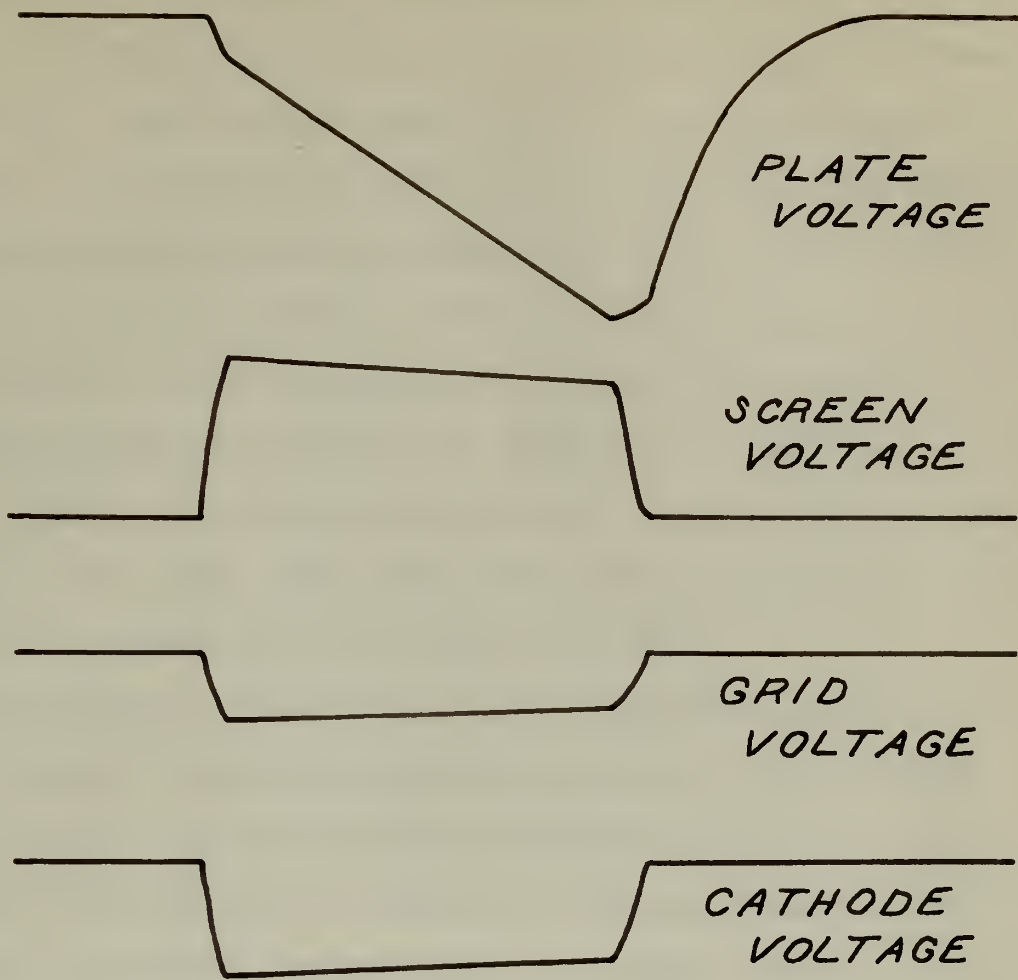
SANAPHANT CIRCUIT

FIGURE 7

space current so that the cathode voltage of V2 drops sharply. This voltage drop is applied to the grid of V1, maintaining V1 cut off, and permitting the linear rundown to occur at the plate of V2. After the plate voltage of V2 bottoms, the grid of V2 begins to rise rapidly toward E_{cc}. The cathode voltage thus rapidly increases by reason of the increased space current in V2. This increase in V_K is applied to the grid of V1, causing V1 to conduct heavily with an abrupt drop in the plate voltage. As with the sanatron, this drop is coupled to the suppressor of V2, cutting off the plate current and ending the sweep action.

Since the cathode voltage of the sanaphant is larger in amplitude and at a much lower output impedance than the grid voltage of the sanatron, the sanaphant exercises more effective control over the grid of the gating pentode.

The output waveforms of the foregoing circuits are identical in shape at corresponding points and are represented in Figure 8. The cathode waveform applies only to the cathode-coupled phantastron and the sanaphant. In practise, the terminating action associated with these waveforms is fast enough for the production of sharp pulses, or for triggering a blocking oscillator. The degree of abruptness in the beginning and at the end of each waveform is, of course, dependent upon the circuit employed and any improvements made thereto. In general, the switching action is faster in the sanatron and the sanaphant than in the cathode-coupled and screen-coupled phantastrons.

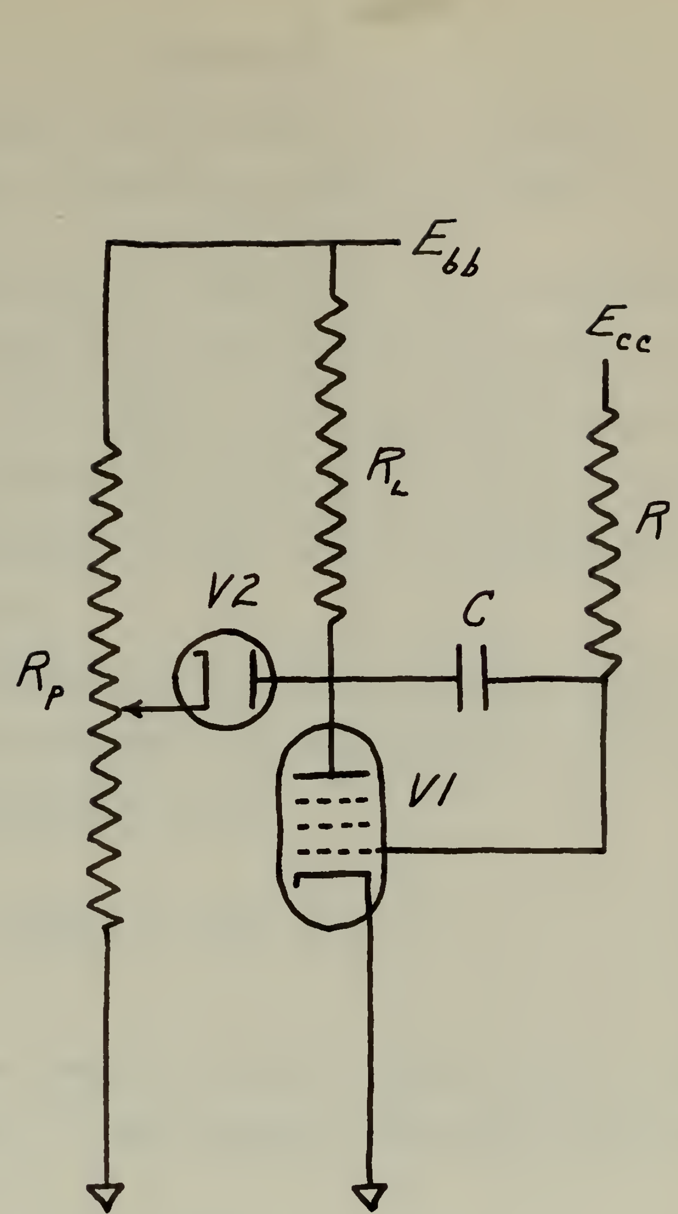


WAVEFORMS OF THE PHANTASTRON, SANATRON, AND SANAPHANT CIRCUITS

FIGURE 8

With this family of circuits, the amplitude of linear time base attainable with a 300 volt power supply is close to 250 volts. Sweep speeds from 1V/sec to 100 V/μsec can be produced. The linearity error can be less than $\pm$ .1 per cent for sweep durations greater than 200 μ sec, and less than $\pm$ .5 per cent for sweep durations from 3 μ sec to 50 μ sec (An increasing error with decreasing RC product is predicted by a linearity equation developed in Chapter I). Variation of the supply voltage by 10 per cent changes the delay time by less than $\pm$ .1 per cent, and a tube replacement changes the delay time also by less than $\pm$ .1 per cent(3,4).

In addition, these circuits are especially suited for use as linear time base generators because, with a slight modification, they can be made to perform the very important function of providing an output discontinuity whose position in time is directly proportional to an input control voltage. This modification is shown in Figure 9. It consists simply of holding the plate voltage of V1 quiescently at that potential selected by the cathode of diode V2. When the amplifier V1 is turned on, however, the plate voltage falls below this control voltage and V2 is an open circuit. But the plate voltage must bottom at the same point regardless of the quiescent plate voltage, and the slope of the plate voltage rundown must remain constant, since E_{cc}, R, and C are constant. Hence, to the extent that the plate voltage sweep is linear, the duration of the waveforms in Figure 8 will be directly proportional to the control voltage setting.



MODIFICATION OF THE MILLER SWEEP GENERATOR TO SELECT THE STARTING POINT OF THE PLATE RUNDOWN

FIGURE 9

CHAPTER IV

IMPROVING THE PERFORMANCE OF CIRCUITS BASED ON THE MILLER SWEEP GENERATOR

It was pointed out in Chapter II that increasing the gain of the amplifier in the Miller sweep generator increases the stability of the circuit, lowers the output impedance at the plate, and improves the linearity of the plate rundown. This may be accomplished by the use of a sizeable inductance in the plate circuit.

If T is the duration of the plate rundown, and Δ E is the change in plate voltage from the beginning of the rundown to the end of the rundown, then

$$\Delta e_b = \Delta E \frac{t}{T} \quad , \tag{25}$$

where Δ e_b is the change in plate voltage from its quiescent value at any time t. Using LaPlace Transform notation,

$$\Delta I_L(s) = \frac{\mathcal{L}e\left[\Delta E \frac{t}{T}\right]}{R_L + Ls} = \frac{\Delta E}{TL} \frac{1}{s^2\left(s + \frac{R_L}{L}\right)} \ . \tag{26}$$

$$\therefore \Delta I_L(t) = \frac{\Delta E}{TL}\left[\frac{e^{-\frac{R_L}{L}t} + \frac{R_L}{L}t - 1}{\frac{R_L^2}{L^2}}\right] . \tag{27}$$

The current at $t = o$ is zero. Hence, the change in current I_L during the sweep is

$$\Delta I_L = \frac{\Delta E}{R_L}\left[1 - \frac{L}{TR_L} + \frac{L}{TR_L} e^{-\frac{R_L}{L}T}\right]$$

$$= \frac{\Delta E}{R_L}\left[1 - \frac{L}{R_L T}\left(1 - e^{-\frac{R_L}{L}T}\right)\right]. \qquad (28)$$

But $\left[1 - \frac{L}{R_L T}\left(1 - e^{-\frac{R_L}{L}T}\right)\right] = 1 - \frac{L}{R_L T}\left(\frac{TR_L}{L} - \frac{T^2 R_L^2}{2L^2} + \cdots\right)$

$$= 1 - 1 + T\frac{R_L}{2L} - \cdots\cdots$$

$$\doteq \frac{TR_L}{2L}, \text{ if } \frac{L}{R_L} \gg T. \qquad (29)$$

$$\therefore \Delta I_L \doteq \frac{\Delta E}{R_L}\left[\frac{T}{2\frac{L}{R_L}}\right]. \qquad (30)$$

With a plate load resistance only, however,

$$\Delta I_{R_L} = \frac{\Delta E}{R_L}. \qquad (31)$$

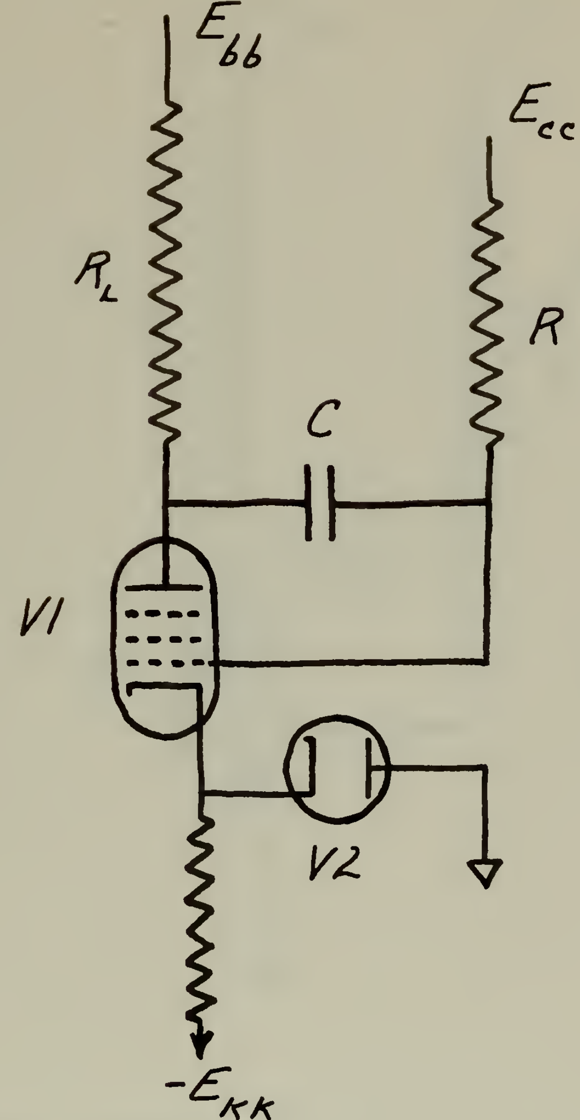
This means that, since $\Delta i_b = \Delta I_L + \Delta I_c = \Delta I_L +$ constant, the presence of a large inductance in the plate circuit results in less plate current change associated with a given plate voltage change Δ E. The constant current characteristics of the amplifier have therefore been bettered, and, with $\frac{\partial i_b}{\partial e_b}$ reduced, the effective gain has been increased. In a specific sanatron circuit, the addition of 20 henries of inductance in the plate circuit boosted the gain of the amplifier from 100 to 4000 and reduced the linearity error to .0006 per cent(3).

With the effective gain of the amplifier thus increased, attention may be directed toward improving the switching action and reducing the switching time. The switch-on action is greatly dependent upon the method of triggering. Obviously the circuits based on the Miller sweep generator can all be triggered at several different points. In general, suppressor triggering is preferred because the triggering can then be isolated from critical biasing circuits. If the trigger has a steep leading edge, application of this trigger to the suppressor will cause the resulting waveforms previously discussed also to have steep leading edges. The trailing edge of the trigger is lost in the internal gating action. But if the trigger is applied to the plate, the possible influence on the plate voltage waveform, and hence on the other waveforms, is apparent. The shape and size of the trigger are then more important.

It frequently happens that the linearity at the beginning

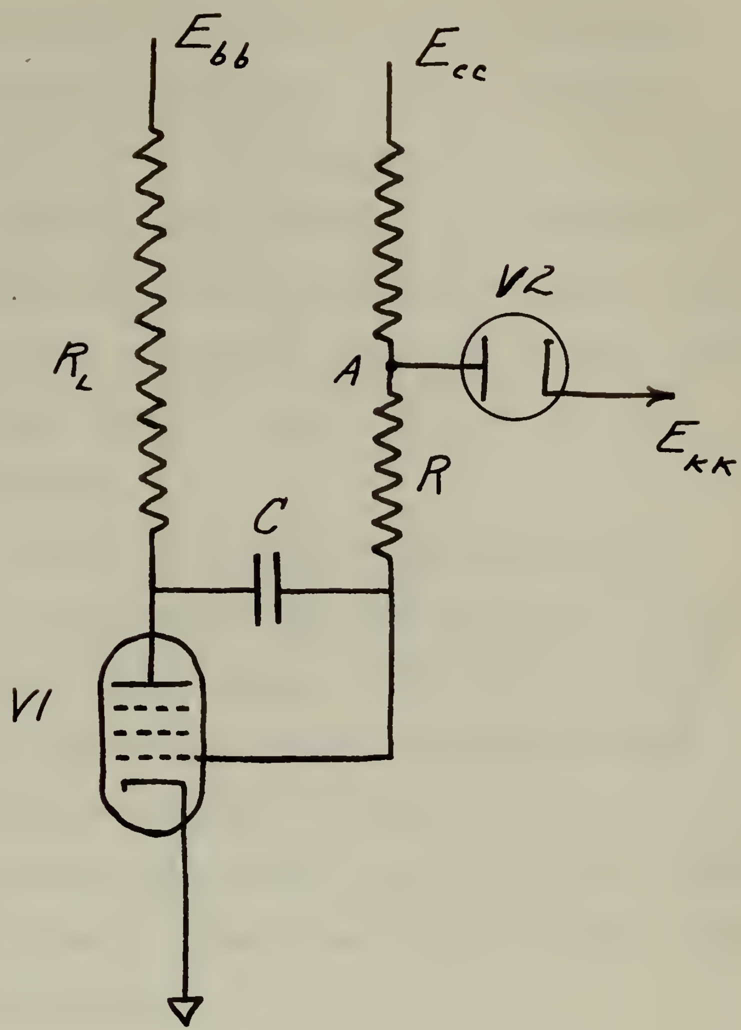
of the rundown is appreciably in error. If the cathode emission of the generator happens to increase momentarily around switch-on time, the grid will be at a potential lower than usual when switch-on occurs. Hence, the grid will drop to a lower than usual potential when the amplifier is turned on and the current through R will momentarily increase. Since $e_b \doteq \frac{E_{cc}}{RC} t$, where $i_R \doteq \frac{E_{cc}}{R}$, the slope of e_b will therefore momentarily increase at the start of the waveform. This error can be compensated for, as shown in Figure 10, by the use of diode V2, where the cathodes of V1 and V2 are heated by the same filament supply. If the cathode of V1 tends to emit more electrons and rise in potential, the cathode of V2 will tend to do the same. But the grounded plate of V2 will repel the electrons and thus hold the cathodes of V1 and V2 at their quiescent potential.

This compensation can also be provided by the circuit of Figure 11, where V2 is connected to a potential E_{kk} lower than E_{cc}. As in the previous method, V1 and V2 must have their cathodes heated by the same filament supply. Quiescently, current is flowing from E_{cc} through a resistor and diode V2 to E_{kk}, and from E_{cc} through the same resistor and R to ground. The current through R is $\frac{E_A}{R}$. Now if the cathode of V1 momentarily emits more electrons, lowering the grid potential by a small increment, the cathode of V2 will simultaneously emit more electrons, lowering the plate of V2 by the same small increment, so that the current through R remains constant.



MODIFICATION OF THE MILLER SWEEP GENERATOR TO COMPENSATE FOR VARIATIONS IN CATHODE EMISSION

FIGURE 10



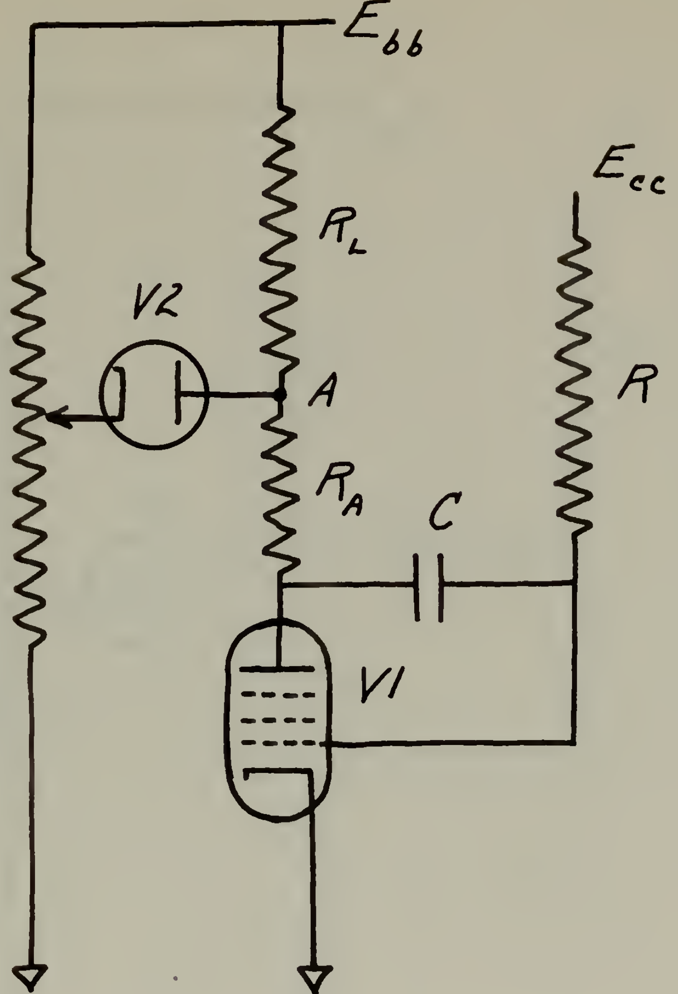
ALTERNATIVE MODIFICATION OF THE MILLER SWEEP GENERATOR TO COMPENSATE FOR VARIATIONS IN CATHODE EMISSION

FIGURE 11

Another characteristic of the generator plate voltage at switch-on time is the initial step in the waveform, which may be undesirable for certain applications, particularly in the case of those circuits employing a cathode resistor. Partial compensation for this step is provided by the circuit of Figure 12. The current flowing through R_L and control voltage diode V2 before switch-on occurs will become a large part of the current drawn through R_L and R_A by V1 when switch-on occurs, so that point A will tend to stay at the same potential at this instant.

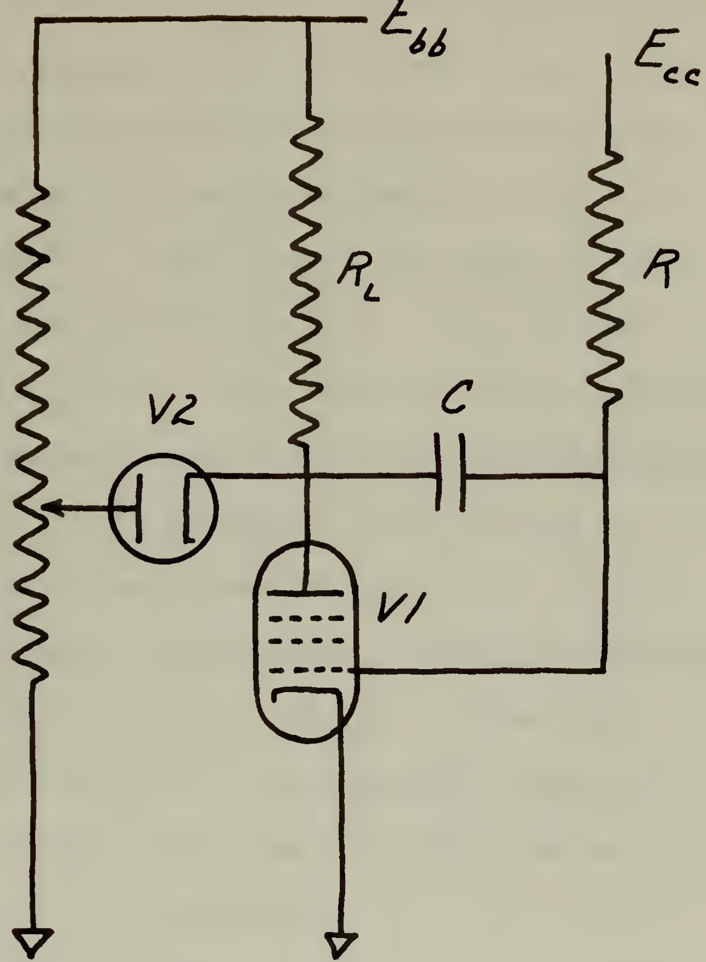
The switch-off action of the generator may also be improved. A technique frequently employed consists of accurately defining the bottoming voltage of the plate rundown at some point above the knee of the pentode characteristics. This may be accomplished by using a diode comparator, as shown in Figure 13, where the minimum value that the plate voltage of the amplifier can have is determined by the voltage setting at the plate of the diode V2.

With the bottoming voltage accurately defined, the switch-off action may be further improved by proper selection of E_{cc}, R, and C. When the rundown bottoms, the grid begins rising toward E_{cc} with time constant RC. The larger E_{cc}, and the smaller RC, the more steeply the grid will rise, and hence the more abruptly it will run into the grid conduction point, at which it is held. In addition, the speed of both the switch-off and switch-on actions may be increased by properly selecting the screen supply voltage. For pentodes,



MODIFICATION OF THE MILLER SWEEP GENERATOR TO COMPENSATE PARTIALLY FOR THE INITIAL STEP IN THE PLATE VOLTAGE

FIGURE 12



MODIFICATION OF THE MILLER SWEEP GENERATOR TO DEFINE A BOTTOMING VOLTAGE ABOVE THE KNEE OF THE $e_b - i_b$ CHARACTERISTICS

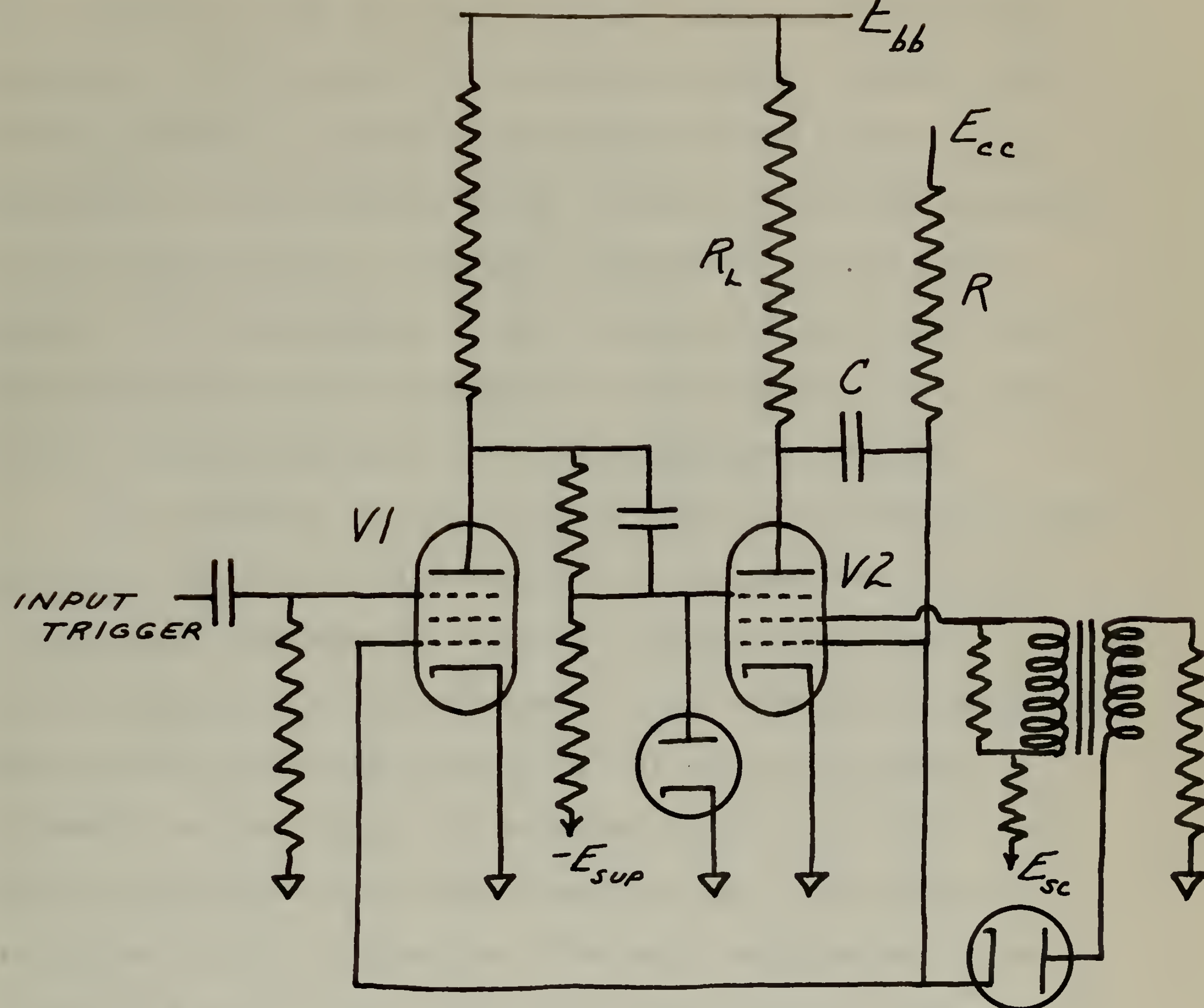
FIGURE 13

a screen voltage of less than 50% of the plate voltage provides the best suppressor control over the plate current(13).

In those circuits where a gating pentode is employed, the slight modification indicated in Figure 14 is very effective in reducing the switch-off time. This modification consists of adding a differentiating transformer between the screen of the Miller sweep generator V2 and the grid of the gating pentode V1. The rise of the grid of V2 at the end of the plate rundown is accompanied by an abrupt fall in the screen voltage of V2. This negative step is differentiated in the transformer primary and the resultant spike voltage is reproduced in the secondary and applied to the grids of both V1 and V2, thus expediting the rise of the grid of V2 to its hold point. The resistor shunting the primary of the transformer must be fairly large, so that the inductance of the primary can pass the low frequency components of the applied step voltage and use the high frequency components for coupling to the secondary.

In the case of the cathode-coupled phantastron and the sanaphant, a similar technique can be used in differentiating the cathode waveform and applying the positive spike resulting from the end of the waveform through a diode to the plate of the generator.

The recovery time of the plate voltage itself can be greatly reduced by driving the grid of a cathode follower directly from the plate of the generator, and returning the capacitance C from the cathode of the follower to the grid



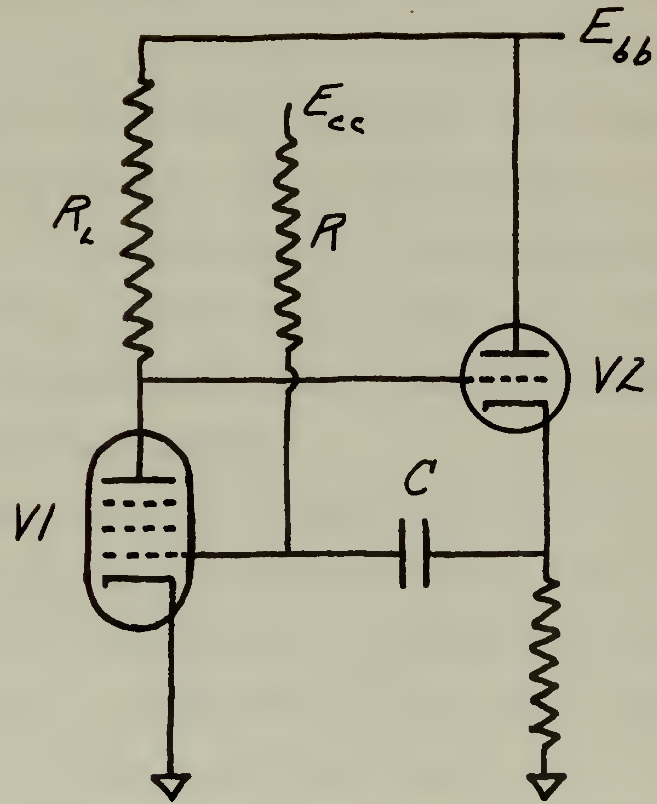
MODIFICATION OF THE SANATRON CIRCUIT TO REDUCE THE SWITCH-OFF TIME

FIGURE 14

of the generator. This basic arrangement is shown in Figure 15. The cathode follower serves as a "unity" gain amplifier, so that the plate voltage of the generator is reproduced at the cathode of the follower and the sweep action proceeds normally. At the end of the linear rundown, however, the time constant of the recovery action at the plate of the generator is the product of R_L and the distributed capacity in the plate circuit, and this product may be made very small. The performance of the cathode follower as a unity gain amplifier can be improved by returning the lower end of the cathode resistor to a large negative voltage.

The operation of the circuits based on the Miller sweep generator may be further improved by temperature compensation of the components. This is a laborious process, consisting of varying the temperature of each component separately while holding the temperature of the other components constant and measuring the resultant effect on the circuit. Such measurements have been completed for a few circuits, contained in the literature. The main requirement in this respect is that R and C have equal and opposite temperature coefficients.

With these circuits modified to include very high gain, fast switching action, and proper compensation, an accurate linear time base of 1 usec duration can be obtained(3,4).



MODIFICATION OF THE MILLER SWEEP GENERATOR TO REDUCE THE RECOVERY TIME OF THE PLATE VOLTAGE

FIGURE 15

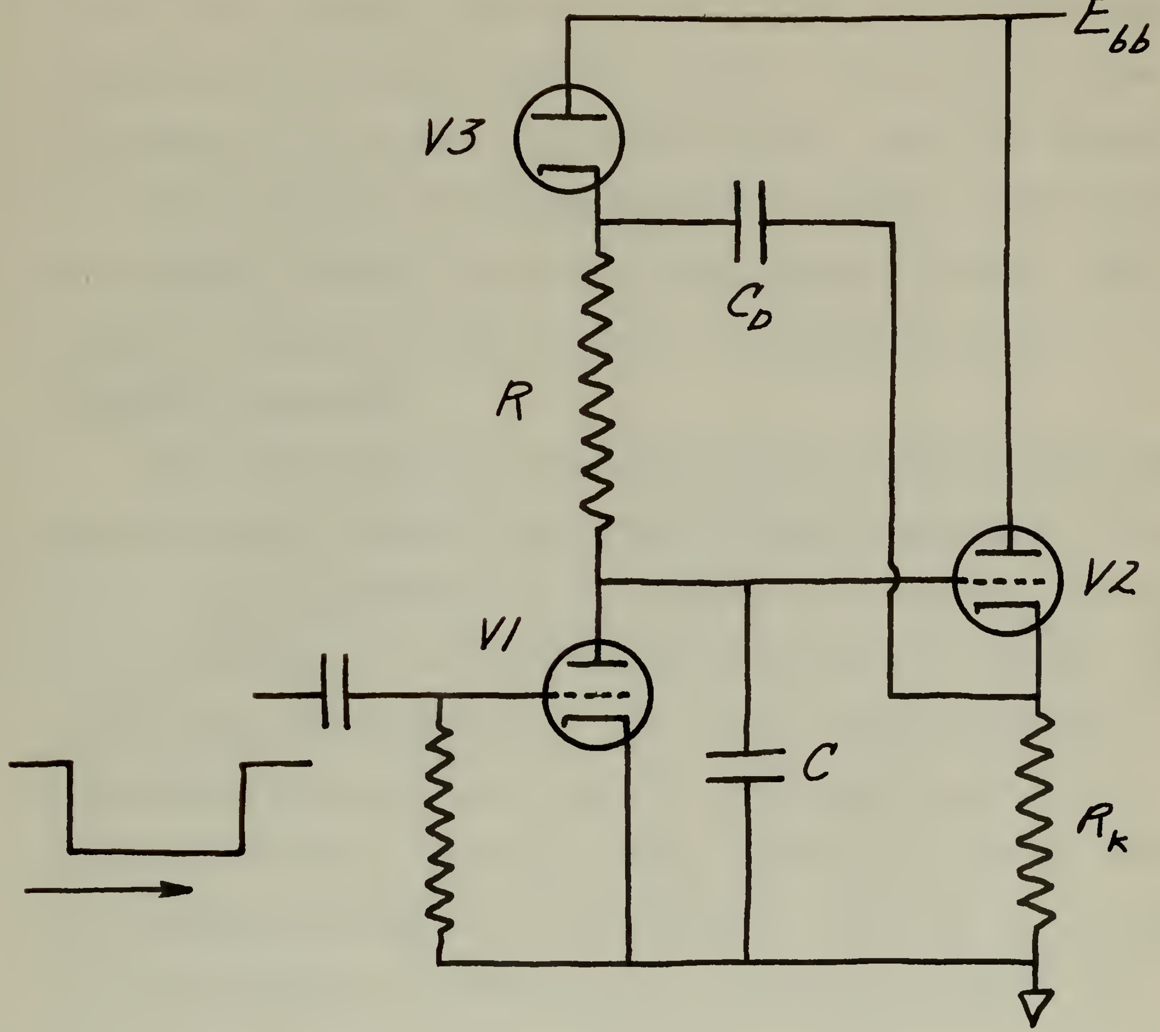
CHAPTER V

COMPARISON OF CIRCUITS BASED ON THE MILLER SWEEP GENERATOR WITH THE BOOTSTRAP CIRCUIT

While the circuits based on the Miller sweep generator were being developed in England, another family of circuits that meet the present-day requirements of linear time base generators, based on the bootstrap principle, was being developed in the United States. Since the operation of the bootstrap circuit and its modifications is more apparent and better understood in this country than the Miller sweep circuits, it will be treated more from a comparison viewpoint rather than in similar detail.

The basic bootstrap circuit is shown in Figure 16. Before the negative input gate is applied to the grid of V1, the grid cathode bias is zero and V1 is conducting heavily with a low plate voltage. Application of the input gate turns V1 off, so that C begins to charge toward E_{bb}. V2 serves as a "unity gain" amplifier, and hence a change in voltage at the plate of V1 appears at the cathode of V2 and is coupled by the large capacitor C_D to the cathode of the isolating diode V3, tending to keep the voltage across R constant. With the voltage across R constant, the charging current for C is constant, and the voltage across C must be a positive-going sawtooth. When the input gate ends, V1 is turned on, and C discharges rapidly through V1.

If the gain of the cathode follower were unity for all points along the sweep, if C_D did not discharge at all



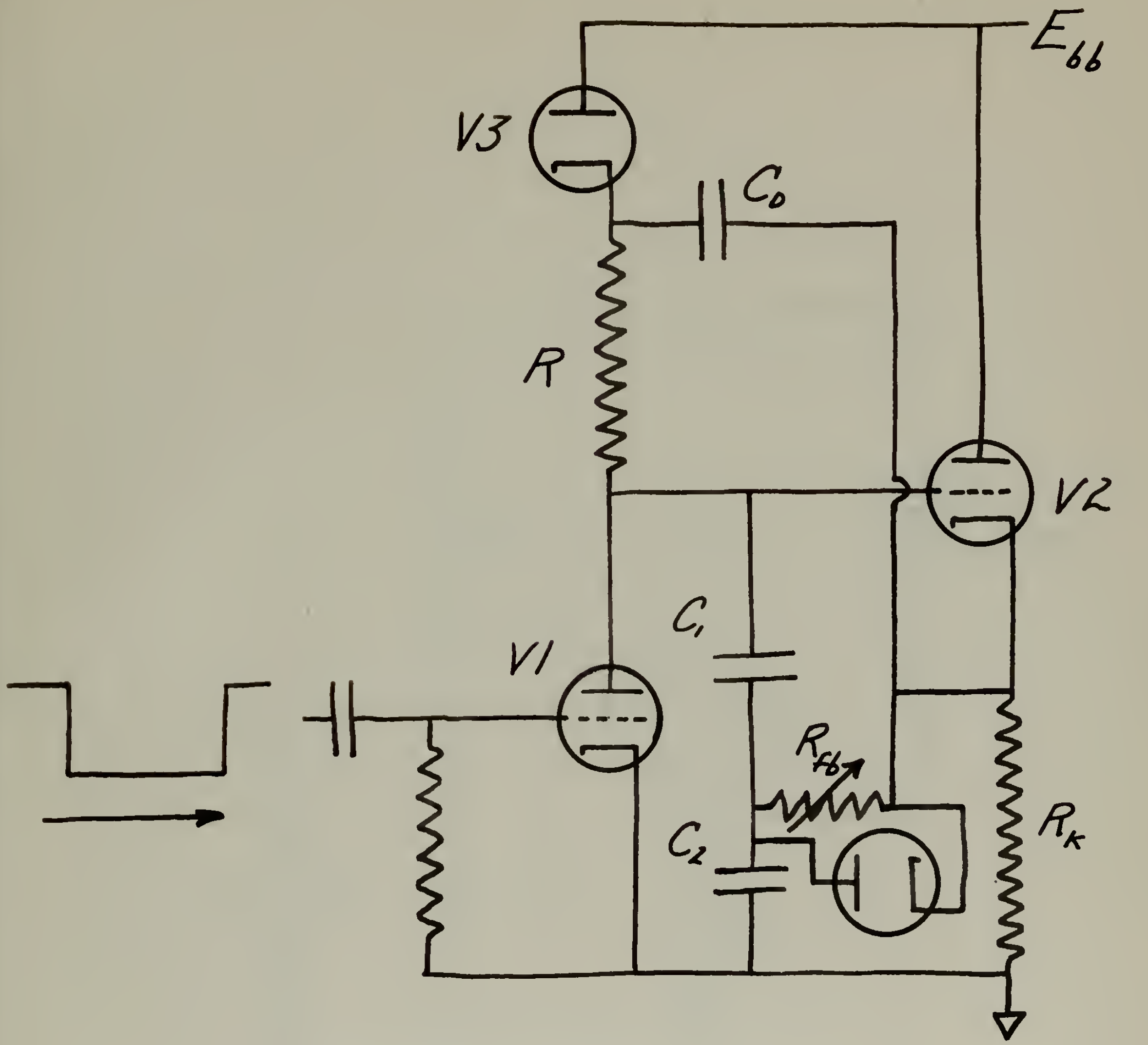
BASIC BOOTSTRAP CIRCUIT

FIGURE 16

during the sweep, and if there were no lead inductance to delay the starting of the sweep, the linearity of the saw-tooth produced would be perfect. But, in practise, the gain of the cathode follower is not unity and it does vary slightly for different input voltages. In addition, some discharge of C_D during the sweep occurs. And lead inductance is always present in varying degrees to impede the build-up of charging current to its required constant value. Some error in linearity with the basic bootstrap circuit is therefore apparent.

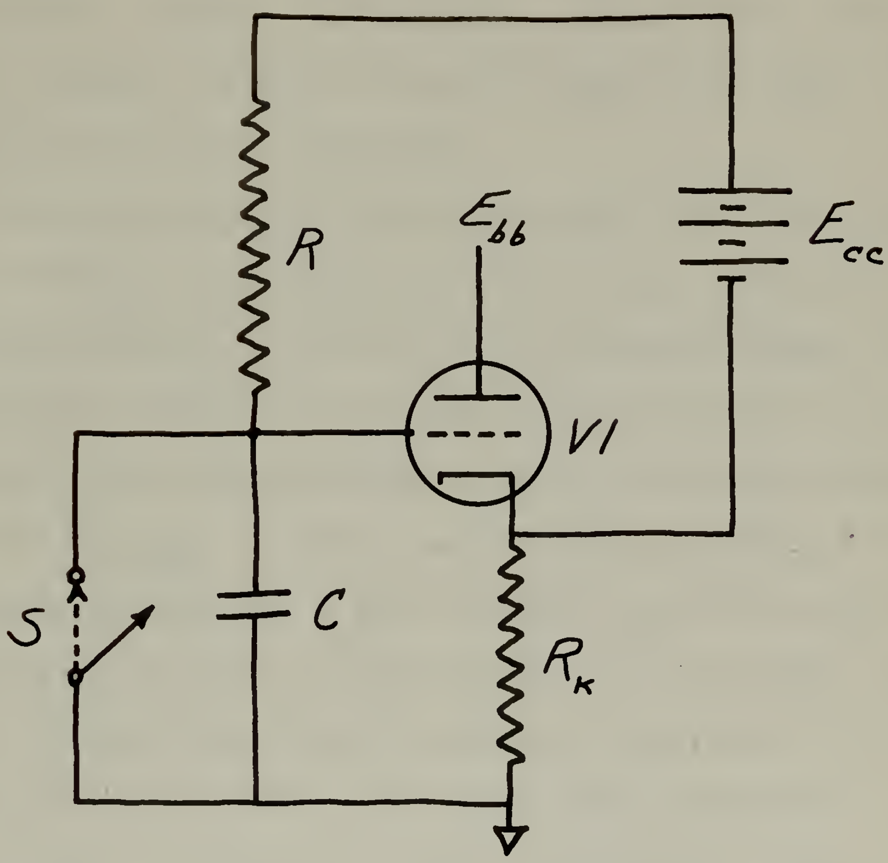
The performance of the bootstrap as a linear time base generator may, however, be substantially improved by certain well known techniques. In the "unity gain" amplifier, R_K may be returned to a large negative voltage to improve the unity gain characteristic. The slight inherent delay in the starting of the sweep due to lead inductance may be compensated for by placing a small resistor in series with the charging capacitor. The linearity may also be improved by the integrator network R_{fb}-C_2 contained in Figure 17. With these modifications, and with C_D sufficiently large, a linearity error of less than .1 per cent is attainable(1,3).

While the bootstrap method of generating a linear time base was developed completely independently from the circuits based on the Miller sweep generator, and appears to operate differently, it should be pointed out at this time that their respective equivalent circuits are almost identical. Figure 18 represents the operation of the bootstrap, where switch



MODIFICATION OF THE BOOTSTRAP CIRCUIT TO IMPROVE THE LINEARITY OF THE SWEEP

FIGURE 17



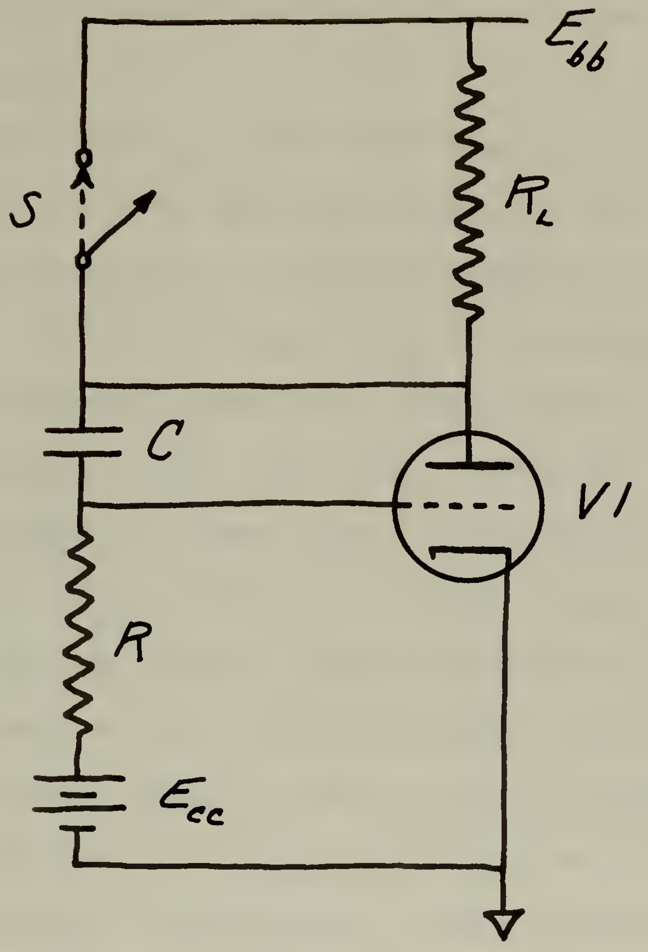
EQUIVALENT CIRCUIT OF THE
BOOTSTRAP CIRCUIT

FIGURE 18

S is the gated triode. Initially, S is closed, as shown in the dotted position. The grid of V1 is at zero potential and the voltage across R is E_{cc}. Opening switch S is equivalent to applying a step voltage E_{cc} to the grid of V1 through R. The charging current for C through R is $\frac{E_{cc}}{R}$. If the bottom of R rises $\Delta\epsilon$ volts, the top of R also rises by $\Delta\epsilon$ volts because of the action of the "unity gain" amplifier V1. Hence, the charging current for C remains $\frac{E_{cc}}{R}$ and the voltage developed across C while S is open is a linear positive-going sawtooth.

Although the Miller sweep generator requires a pentode, we can represent its amplifier action by a triode and the switching process by a switch S, as shown in Figure 19. Quiescently, switch S is closed, the grid of V1 is at zero potential, the voltage across R is E_{cc}, and the current through R is $\frac{E_{cc}}{R}$. As with the bootstrap, opening S is equivalent to applying a step voltage E_{cc} to the grid of V1 through R and turning on the amplifier at the same time. Because of the drop in plate voltage coincident with the turn-on of the amplifier, the grid drops below zero. But it was shown in Chapter II that the amplifier tends to hold the grid at a negative voltage close to zero. Hence, the current to C remains approximately $\frac{E_{cc}}{R}$ while the switch is open, the same as with the bootstrap. In this case, however, the constant current to C is a discharging current, and the voltage across C is a linear negative-going sawtooth.

In performing the important function of generating a

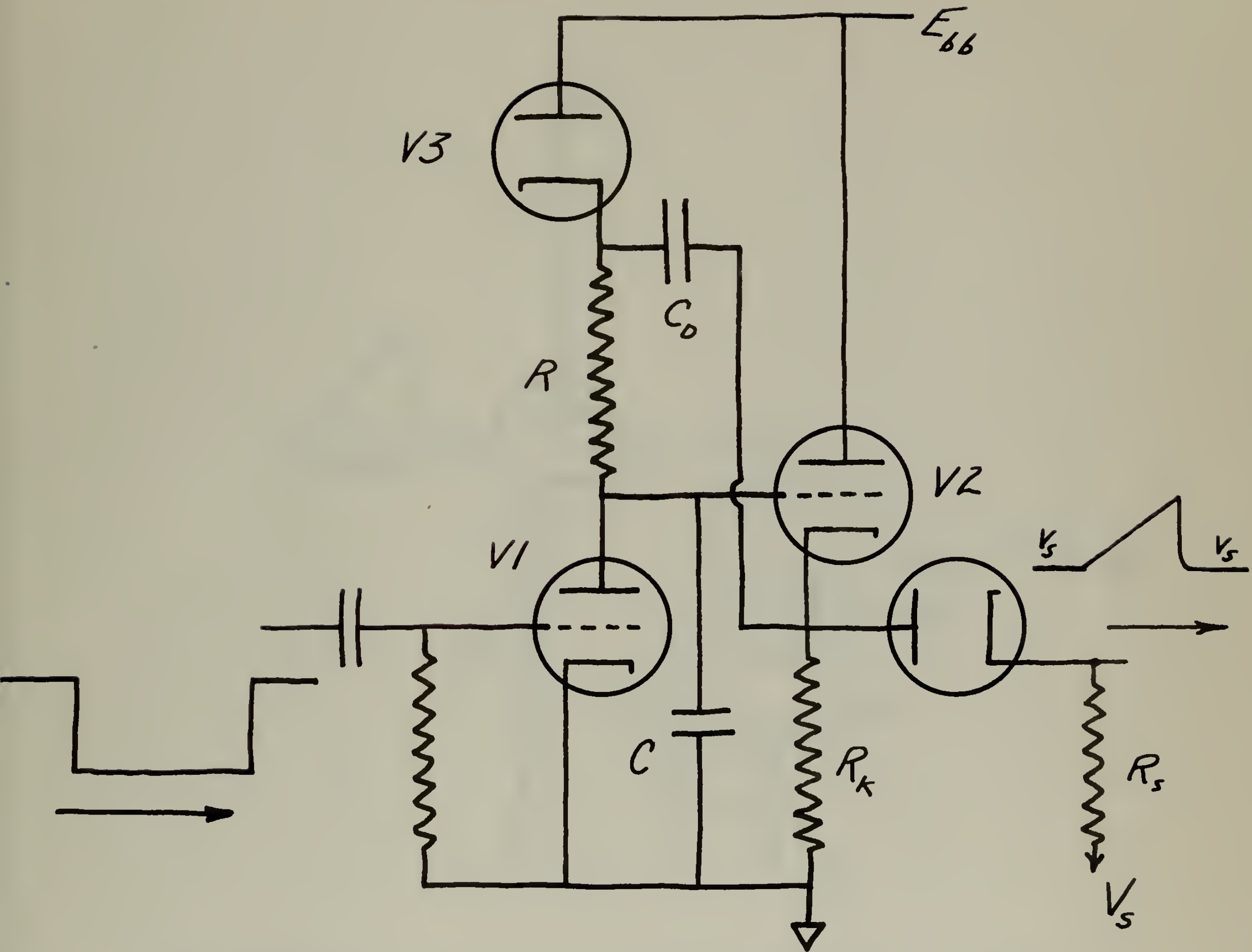


EQUIVALENT CIRCUIT OF THE
MILLER SWEEP GENERATOR

FIGURE 19

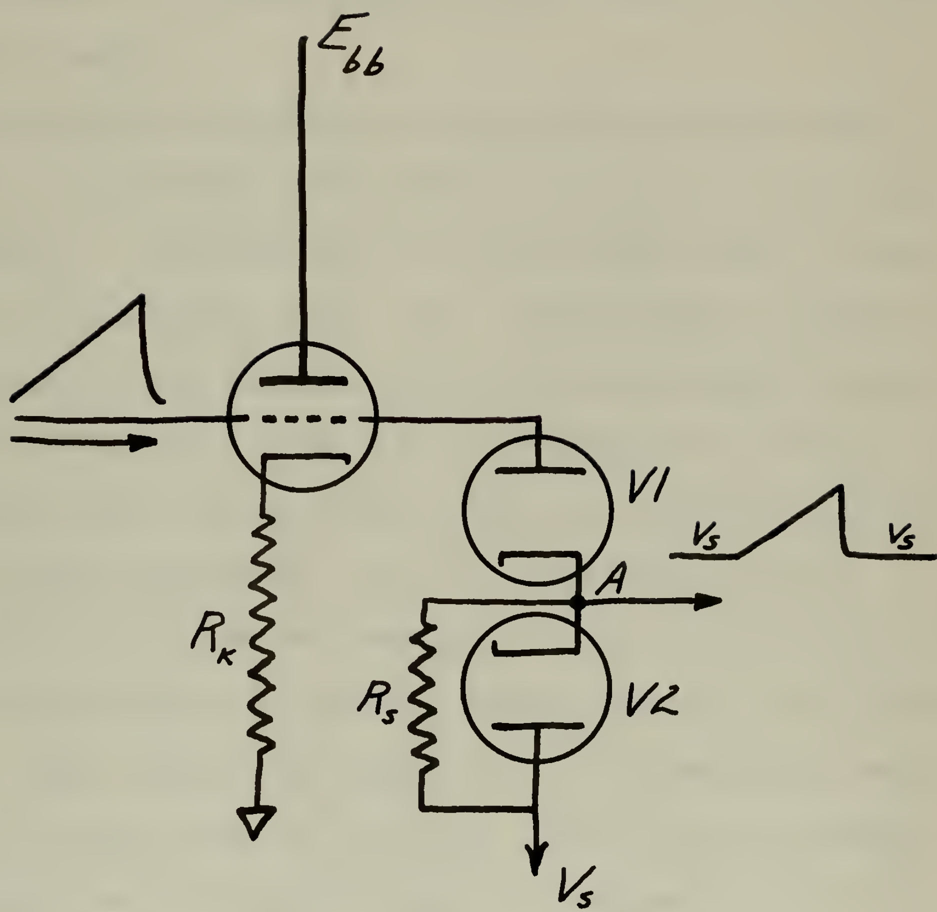
discontinuity whose position in time is directly proportional to a control voltage, the circuits based on the Miller sweep generator have one disadvantage. There is no known means of compensating for slight variations in emission from the cathode of the control voltage diode, so that a slight zero error is inherent in these circuits. This error has been estimated at around .05 per cent for the average sweep duration(3).

An advantage of the bootstrap as a linear time base generator is that this zero error at the pickoff point may be compensated for, as shown in Figures 20 and 21. In this case the control voltage V_s does not determine the point where the sweep begins, but merely selects that portion of the generated sweep which lies above V_s. In Figure 20, where pickoff is taken from the cathode of the cathode follower, if the potential of the cathode of the pickoff diode tends to change from fluctuations in heater voltage, the cathode of the cathode follower, when heated from the same filament supply, tends to drift along, so that pickoff occurs at about the same point in time as if no fluctuations had occurred. In Figure 21, where pickoff is taken from the grid of the cathode follower, the control voltage circuit exactly compensates for variations in the heater voltage of the pickoff diode V1. For voltages along the sawtooth below V_s, point A is strapped to V_s. But beyond the point of selection, point A is above V_s, V2 is an open circuit, and the remainder of the sawtooth appears across R_s. In applying



INHERENT COMPENSATION IN THE BOOTSTRAP CIRCUIT FOR VARIATIONS IN EMISSION OF THE PICKOFF DIODE, FOR PICKOFF AT THE CATHODE

FIGURE 20



COMPENSATION IN THE BOOTSTRAP CIRCUIT FOR VARIATIONS IN CATHODE EMISSION OF THE PICKOFF DIODE, FOR PICKOFF AT THE GRID

FIGURE 21

these control voltage circuits, however, it should be noted that in Figure 20 the output impedance of the cathode follower is low and the size of R_s is not of great importance, whereas in Figure 21 the output impedance at the grid of the cathode follower is theoretically $\frac{1}{C}$, and hence in this case R_s should be large.

The effectiveness of this compensation depends upon which end of the picked-off sawtooth is used for the output discontinuity. The duration of the bootstrap sweep is the duration of the applied gate, and, if the gating circuit is a multivibrator, the terminal point of the bootstrap sweep and hence the terminal point of the picked-off sawtooth may vary far more than the zero-time point of circuits based on the Miller sweep generator. On the other hand, if the beginning of the picked off sawtooth is used with a circuit such as the multiar to generate a discontinuity at the pick-off point, then, since the beginning of the bootstrap sweep occurs at the same time as the input trigger to the gating system, and since the pickoff point is compensated for the possible variations discussed, the discontinuity will occur at an accurately defined point in time directly proportional to the control voltage V_s.

A disadvantage of the bootstrap circuit is that the isolating diode V3 (Figure 16) requires an isolated heater supply voltage with one side strapped to the cathode of V3, for in practise the positive feedback sawtooth voltage from the cathode of V2 to the cathode of V3 would otherwise

result in heater-cathode breakdown in V3 at some point along the sweep. Another disadvantage of the bootstrap circuit, from a space and cost viewpoint, is the large number of tubes required for the system.

CHAPTER VI

CONCLUSIONS

Chapter II points out that the basic Miller sweep generator produces a highly linear negative-going sawtooth voltage waveform at a low output impedance, with good stability. It is shown in this chapter how with increasing gain the stability and linearity are improved and the output impedance is lowered. This analysis serves as a basis for the development in Chapter III of a family of circuits comprising screen-coupled and cathode-coupled phantastrons, the sanatron, and the sanaphant, all of which can be easily modified to perform the very important function of providing an output discontinuity whose position in time is directly proportional to an input control voltage. Each of these circuits represents a basic Miller sweep generator, internally gated by some means which distinguishes the circuits from each other. Those circuits employing a cathode resistor make available four output voltages at both low and high output impedances. Those circuits utilizing a grounded cathode make available three output voltages, also at both low and high output impedances. These voltages at corresponding points in all the circuits are similar in form, and in any circuit the available output voltages are characterized by the same time delay. For a 300 volt power supply, the amplitude of sawtooth voltage at the plate of the generator in each case can be close to 250 volts, and sweep speeds from 1V/sec to 100V/usec can be produced. The linearity error can

be less than $\pm$ .1 per cent for sweep durations greater than 200 usec, and less than $\pm$ .5 per cent for sweep durations from 3 usec to 50 usec. Variation of the supply voltage by 10 per cent changes the delay time by less than $\pm$ .1 per cent, and a tube replacement changes the delay time also by less than $\pm$ .1 per cent(3,4).

In Chapter IV the analysis is extended to show how the gain of these generators may be substantially increased by inserting a large inductance in series with the plate load resistor. In one specific case, the addition of 20 henries of inductance in the plate circuit boosted the gain of the amplifier from 100 to 4000 and reduced the linearity error to .0006 per cent(3). It is also shown how the performance may be improved with various modifications of the basic circuits and with compensating networks, to the extent that an accurate linear time base of 1 usec duration can be obtained(3,4).

It is subsequently brought out in Chapter V that the circuits based on the Miller sweep generator have one disadvantage in that there is no known means of compensating for slight variations in emission from the cathode of the control voltage diode. When a bootstrap circuit is used to provide an output discontinuity whose position in time is directly proportional to a control voltage, however, the zero error at the pickoff point due to variations in emission from the cathode of the control voltage diode may be readily compensated for. A further comparison of the

two types of circuits is effected wherein the advantages and disadvantages of the bootstrap are outlined. With certain modifications to the basic bootstrap circuit, a positive-going sawtooth voltage with a linearity error of less than .1 per cent is attainable(1,3). Two output voltages are available: at the cathode of the cathode follower, with a low output impedance, and at the grid of the cathode follower, with a high output impedance. The circuit is simple and its operation easily understood. But in practise the isolating diode in the bootstrap circuit requires an isolated heater supply voltage with one side strapped to the cathode of the diode, to avoid heater-cathode breakdown in this tube as the sweep progresses. In addition, since this system consists of a gate generator switching tube, cathode follower, isolating diode, and control voltage diode, it requires a relatively large number of tubes.

With regard to stability, amplitude of sweep voltage, range of sweep speeds, and insensitivity to tube changes and variations in power supply voltage, both types of circuits are relatively equal in performance. It should be pointed out, however, that in the bootstrap circuit the linearity error tends to become proportionately greater with long sweep durations because of the discharge of the large coupling capacitor between the cathode of the cathode follower and the cathode of the isolating diode.

From a space and cost viewpoint, the phantastron, sanatron, and sanaphant circuits have a distinct advantage

over the bootstrap circuit, and since the zero error inherent in circuits based on the Miller sweep generator is almost negligible, these circuits are generally preferred for linear time base applications(3).

BIBLIOGRAPHY

1. Close, R.N., and M.T. Lebenbaum. Design of phantastron time delay circuits. Electronics. 21:100, April 1948.

2. Chance, B. Some precision circuit techniques used in waveform generation and time measurement. Review of Scientific Instruments. 17:396, October, 1946.

3. Chance, B., V. Hughes, E.F. MacNichol, D. Sayre, and F.C. Williams. Waveforms. New York, McGraw-Hill. 1949. (Massachusetts Institute of Technology. Radiation laboratory series. No. 19)

4. Chance, B., R. I. Hulsizer, E. F. MacNichol, and F. C. Williams. Electronic time measurements. New York, McGraw-Hill, 1949. (Massachusetts Institute of Technology. Radiation laboratory series. No. 20)

5. Greenwood, I. A., J. V. Holdam, Jr., and D. MaCrae, Jr. Electronic instruments. New York, McGraw-Hill. 1948. (Massachusetts Institute of Technology. Radiation laboratory series. No. 21)

6. Handbook of maintenance instructions for Model AN/APS-15 aircraft radar equipment, CO-AN-08.30 APS15-2. U. S. Navy.

7. Holdam, J. V., S. McGrath, and A. D. Cole. Radar for blind bombing. Electronics. 19:138, May, 1946.

8. McDade, J. R. The phantastron control circuit. Electrical Engineering. 67:974, October, 1948.

9. Medley, D. J., and H. D. Rathgeber. A precision double-pulse generator. Journal of Scientific Instruments. 25:234, July 1948.

10. Ridenour, Louis N. Radar system engineering. New York, McGraw-Hill. 1947. (Massachusetts Institute of Technology. Radiation laboratory series. No. 1)

11. Seely, Samuel. Electron tube circuits. New York, McGraw-Hill. 1950.

12. Snowden, S. C. Analysis of a Multivibrator. California Institute of Technology. 1945.

13. Spangenberg, K. R. Vacuum tubes. New York, McGraw-Hill. 1948.

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